12 Surface Areas and Volumes
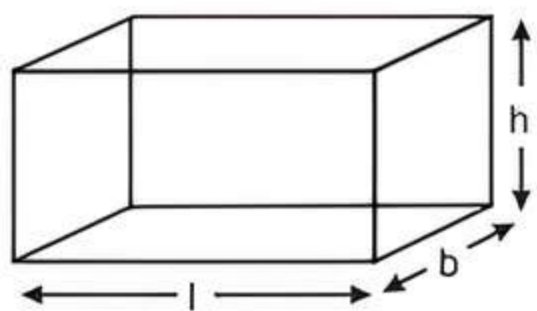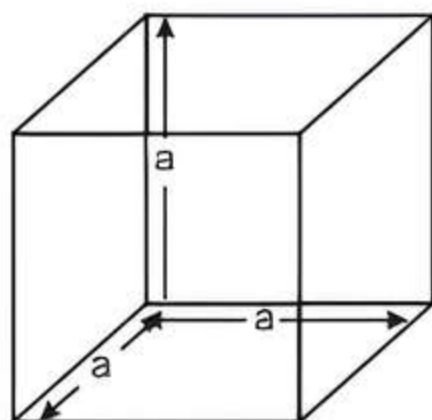
▶ **Cuboid:** A cuboid has six faces. Let the length of a cuboid be 'l', breadth be 'b' and height be 'h'.

1. Volume $= l \times b \times h$ cubic units
2. Lateral Surface Area (LSA)
   $=$ Area of the four walls of a room
   $= 2(l + b) \times h$ square units
3. Total Surface Area (TSA) $= 2(lb + bh + hl)$ square units
4. Length of the diagonal $= \sqrt{l^2 + b^2 + h^2}$ units

▶ **Cube:** A cube has six identical faces. Let each edge of a cube be 'a'.

1. Volume $= (\text{Side})^3 = a^3$ cubic units
2. Lateral Surface Area (LSA) $= 4a^2$ square units
3. Total Surface Area (TSA) $= 6a^2$ square units
4. Diagonal $= a\sqrt{3}$ units

▶ **Right Circular Cylinder:** A cylinder has one curved face and two plane circular faces. Let 'r' be the base radius and 'h' be the vertical height of a cylinder.

1. Volume $=$ Area of base $\times$ Height
   $= \pi r^2 h$ cubic units
2. Curved Surface Area (CSA)
   $=$ Circumference of base $\times$ Height
   $= 2\pi rh$ square units
3. Total Surface Area (TSA) $=$ CSA + Area of two ends
 $= 2\pi rh + 2\pi r^2 = 2\pi r(h + r)$ square units
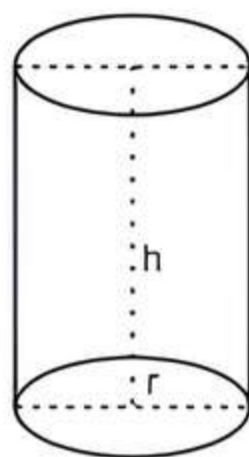
▶ **Hollow Cylinder:** Solids like pipes are in the shape of hollow cylinders. Let 'R' be the external radius, 'r' be the internal radius and 'h' be the height of a hollow cylinder.

1. Area of cross-section
   $= \pi(R^2 - r^2)$ square units
2. Volume of material
   $= \pi R^2 h - \pi r^2 h$
   $= \pi h(R^2 - r^2)$
   $= \pi h(R + r)(R - r)$ cubic units
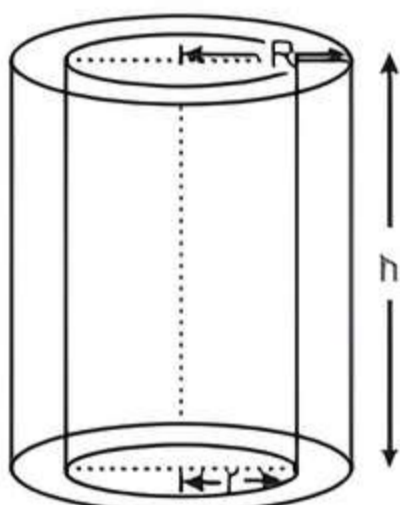
3. Curved Surface Area (CSA) = External surface area + Internal surface area
   $= 2\pi Rh + 2\pi rh = 2\pi h(R + r)$ square units
4. Total Surface Area (TSA)
   $=$ Curved surface area of hollow cylinder + Area of top and base rings
   $= 2\pi h(R + r) + 2\pi(R^2 - r^2)$
   $= 2\pi (R + r)(R - r + h)$ square units
5. Thickness of cylinder $= R - r$ units

▶ **Right Circular Cone:** A cone has one curved face and one plane circular face. Solids like Joker's cap, funnel, softy ice-cream, etc. are in the shape of cone. Let 'r' be the radius, 'h' be the vertical height and 'l' be the slant height of a right circular cone.
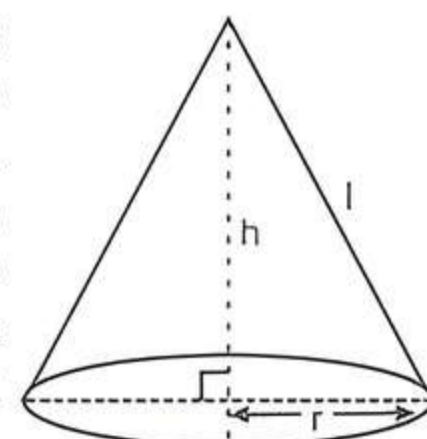
1. Volume $= \frac{1}{3}\pi r^2 h$ cubic units
2. Curved Surface Area (CSA) $= \pi r l$ square units
3. Total Surface Area (TSA)
   $=$ CSA + Area of the base
   $= \pi r l + \pi r^2$
   $= \pi r(l + r)$ square units
4. Slant height $(l) = \sqrt{h^2 + r^2}$ units

▶ **Sphere:** A sphere has only one curved surface. Let 'r' be the radius of sphere.

1. Volume $= \frac{4}{3}\pi r^3$ cubic units
2. Curved Surface Area (CSA)
   $=$ Total Surface Area (TSA) $= 4\pi r^2$ square units
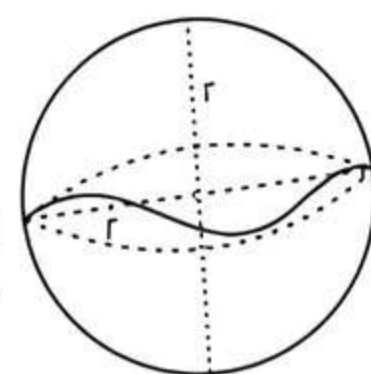
▶ **Spherical Shell (Hollow Sphere):** Let 'R' be the external radius and 'r' be the internal radius.

1. Volume of material
   $=$ Outer volume $-$ Inner volume
   $= \frac{4}{3}\pi R^3 - \frac{4}{3}\pi r^3$
   $= \frac{4}{3}\pi(R^3 - r^3)$ cubic units

2. Thickness $= (R - r)$ units
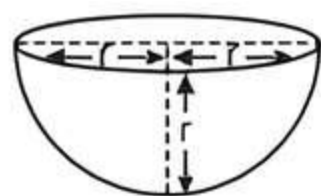3. Outer surface area $= 4\pi R^2$
4. Inner surface area $= 4\pi r^2$
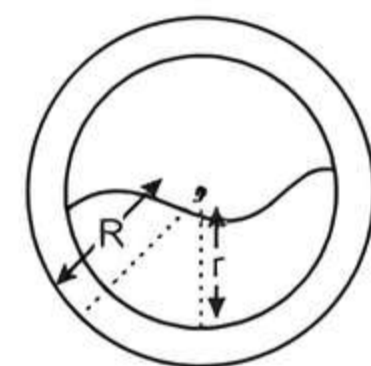
▶ **Hemisphere:** A plane through the centre of a sphere cuts it into two equal parts. Each part is called a hemisphere. Let 'r' be the radius of hemisphere.

1. Volume $= \frac{2}{3}\pi r^3$ cubic units

2. Curved Surface Area (CSA)
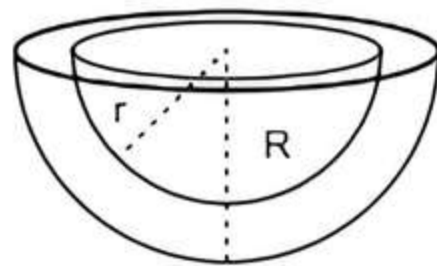 $= 2\pi r^2$ square units
3. Total Surface Area (TSA)
   = CSA + Area of the base
   $= 2\pi r^2 + \pi r^2 = 3\pi r^2$ square units

▶ **Hemispherical Shell:** Let 'R' be the external radius and 'r' be the internal radius.

1. Area of base $= \pi(R^2 - r^2)$ square units
2. Volume of material
   $= \frac{2}{3}\pi(R^3 - r^3)$ square units

▶ **Surface Area and Volume of Combination of Solid Figures:** Supposed combined figure of hemisphere and cylinder is given below:

1. The surface area of combined figure is the sum of all areas that are visible, (*i.e.*, generally the sum of all the curved surface of the individual figure.)
   Surface area of combined figure
   = CSA of hemisphere + CSA of cylinder
   + CSA of hemisphere.
2. The volume of combined figure is the sum of all the volumes of the individual figure.

Volume of combined figure
   = Volume of hemisphere + Volume of cylinder
   + Volume of hemisphere.

Knowledge BOOSTER

1. *The maximum length of rod that can be fitted in a cuboid (or cube) is equal to the length of diagonal.*
2. *When we adjoin the two or more cubes, only the length of cuboid changes, but breadth and height remain constant.*
3. *Cost = Area × Rate*
4. *Density* $= \dfrac{Mass}{Volume}$
5. **Important Conversions of Units**
   - $1\ m^3 = 1000\ L$
   - $1\ m^3 = 1\ kL$
   - $1\ L = 1000\ cm^3$
   - $1\ mL = 1\ cm^3$
   - $1\ mm = \frac{1}{10}\ cm$
   - $1\ km/hr = \frac{5}{18}\ m/s$
   - $1\ m/s = \frac{18}{5}\ km/hr$
   - $1\ hectare = 10{,}000\ m^2$

Practice Exercise

Multiple Choice Questions

Q 1. The shape of a *gilli*, in the *gilli-danda* game is a combination of: *[NCERT EXEMPLAR]*
   a. two cylinders
   b. one cone and one cylinder
   c. two cones and one cylinder
   d. two cylinders and one cone

Q 2. A Surahi is the combination of: *[NCERT EXEMPLAR]*
   a. a sphere and a cylinder
   b. a hemisphere and a cylinder
   c. two hemispheres
   d. a cylinder and a cone

Q 3. A cylinder and a cone have same base and same height. The ratio of their volumes is:
   a. 3 : 1   b. 1 : 3   c. 2 : 3   d. 3 : 2

Q 4. If two cubes of edge 3 cm each are joined end to end, then the surface area of the resulting cuboid is:
   a. 90 cm^2 b. 95 cm^2 c. 92 cm^2 d. 94 cm^2

Q 5. What is the total surface area of a solid hemisphere of diameter 'd'? *[CBSE 2023]*
   a. $3\pi d^2$ b. $2\pi d^2$ c. $\frac{1}{2}\pi d^2$ d. $\frac{3}{4}\pi d^2$

Q 6. If two solid hemispheres of same base radius r are joined together along their bases, then curved surface area of this new solid is: *[NCERT EXEMPLAR]*
   a. $3\pi r^2$ b. $6\pi r^2$ c. $3\pi r^2$ d. $8\pi r^2$

Q 7. Suppose h and r be the height and radius of cylinder. If cylinder is cut from the middle part, then the volume of the remaining part of cylinder is:
   a. $\pi r^2 h$ b. $\dfrac{\pi r^2 h}{2}$
   c. $2\pi r^2 h$ d. None of these

Q 8. The curved surface area of a right circular cylinder of hight 14 cm is 88 cm^2. The diameter of its circular base is: *[CBSE 2023]*
   a. 2 cm   b. 1 cm   c. 4 cm   d. 7 cm

Q 9. The volume of a cone of radius 'r' and height '$3r$' is: *[CBSE 2023]*
   a. $\frac{1}{3}\pi r^3$ b. $3\pi r^3$
   c. $9\pi r^3$ d. πr^3

Q 10. The curved surface area of a cone of radius 7 cm is 550 cm^2. Its slant height is: *[CBSE 2023]*
   a. 24 cm   b. 25 cm   c. 14 cm   d. 20 cm

Q 11. The volume of a right circular cone whose area of the base is 156 cm^2 and the vertical height is 8 cm, is: *[CBSE 2023]*
   a. 2496 cm^3 b. 1248 cm^3
   c. 1664 cm^3 d. 416 cm^3

Q 12. Suppose height and radius of a solid cylinder are 15 cm and 6 cm. A cone is carved out from a cylinder the maximum height of cone is:
   a. 6 cm   b. 15 cm   c. 13 cm   d. 17 cm

Q 13. The volumes of two spheres are in the ratio 64 : 27. The ratio of their surface areas is:

a. 4 : 3   b. 16 : 9   c. 9 : 16   d. 2 : 3

Q 14. The ratio of lateral surface area to the total surface area of a cylinder with base diameter 1.6 m height 20 cm is:

a. 1 : 5   b. 5 : 1   c. 3 : 2   d. 2 : 3

Q 15. The radii of two cylinders are in the ratio 3 : 5. If their heights are in the ratio 2 : 3, then the ratio of their curved surface areas is:

a. 2 : 5   b. 5 : 2   c. 3 : 4   d. 4 : 3

Q 16. A solid ball is exactly fitted inside the cubical box of side 2 cm. The volume of the ball is:

a. $\frac{16}{3}\pi$ cm^3

b. $\frac{4}{3}\pi$ cm^3

c. $\frac{33}{2}\pi$ cm^3

d. None of these

Q 17. A solid cylinder of radius R and height H is placed other cylinder of same height and radius. The total surface area of the shape so formed is:

a. $2\pi R(R + 3H)$

b. $2\pi R(R + 2H)$

c. $2\pi R(R - 2H)$

d. None of these

Q 18. A rectangular sheet of paper 40 cm × 22 cm, is rolled to form a hollow cylinder of height 40 cm. The radius of the cylinder (in cm) is:

[CBSE SQP 2023-24]
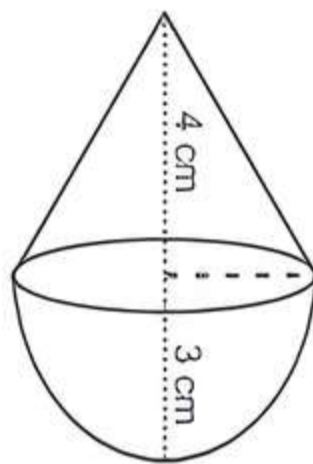
a. 3.5   b. 7   c. $\frac{80}{7}$ d. 5

Q 19. A solid is of the form of a cone of radius 'r' surmounted on a hemisphere of the same radius. If the height of the cone is the same as the diameter of its base, then the volume of the solid is:

[CBSE 2023]

a. πr^3

b. $\frac{4}{3}\pi r^3$

c. $3\pi r^3$

d. $\frac{2}{3}\pi r^3$

Q 20. The surface area of the following figure is:

a. 20π cm^2 b. 33π cm^2 c. 18π cm^2 d. 17π cm^2

Assertion & Reason Type Questions

Directions (Q. Nos. 21-25): *In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)

c. Assertion (A) is true but Reason (R) is false

d. Assertion (A) is false but Reason (R) is true

Q 21. Assertion (A): Suppose two equal cubes of edge 4 cm are joined together then the surface area of resulting cuboid is 160 cm^2.

Reason (R): We combined two equal cubes of edge a cm, then the length of the resulting cuboid will be $2a$ cm.

Q 22. Assertion (A): The radii of two cones are in the ratio 2 : 3 and their volumes in the ratio 1 : 3. Then the ratio of their heights is 3 : 4.

Reason (R): Volume of a cone can be determined by the formula, $V = \frac{1}{3}\pi r^2 h$.

Q 23. Assertion (A): Total surface area of the toy is the sum of the curved surface area of the hemisphere and the curved surface area of the cone.

Reason (R): Toy is obtained by fixing the plane surfaces of the hemisphere and cone together.
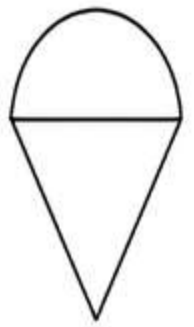
[CBSE SQP 2023-24]

Q 24. Assertion (A): If the radius of a cone is halved and volume is not changed, then height remains same.

Reason (R): If the radius of a cone is halved and volume is not changed then height must become four times of the original height.

Q 25. Assertion (A): If the volumes of two spheres are in the ratio 216 : 125. Then their surface areas are in the ratio 6 : 5.

Reason (R): Volume of the sphere $= \frac{4}{3}\pi r^3$ and its surface area $= 4\pi r^2$.

Fill in the Blanks Type Questions

Q 26. The maximum length of rod that can be placed in the cuboid box is equal to its ....

Q 27. If a sphere and a cube have equal surface areas, then the ratio of the volume of the sphere to that of the cube is ....

Q 28. The surface area of a sphere whose volume is 4851 cubic metres is ....

Q 29. If the areas of three adjacent faces of cuboid are x, y, z respectively, then the volume of the cuboid is ....

Q 30. If R and r be the external and internal radius of a hemispherical shell, then the capacity of the hemispherical shell is .... square units.

💡 True/False Type Questions ↘

Q 31. The volume of the material in a hollow body is equal to the difference between the external volume and internal volume.

Q 32. The maximum volume of a cone that can be carved out of a solid hemisphere of radius r is $\frac{1}{2}\pi r^2$.

Q 33. The ratio between the volumes of two spheres is 8 : 27, the ratio between their surface areas is 4 : 9.

Q 34. A cube is a special type of cuboid.

Q 35. If we double the radius of a hemisphere, its surface area will also be doubled.

Solutions

1. (c) two cones and one cylinder
2. (a) a sphere and a cylinder
3. (a) Let r and h be the same radii and same height of cylinder and a cone.
   $\therefore$ Volume of cylinder : Volume of cone
   $$= \pi r^2 h : \frac{1}{3}\pi r^2 h = 1 : \frac{1}{3} = 3 : 1$$
4. (a) Given, edge of a cube is 3 cm.

   3 cm ⊢ 3 cm
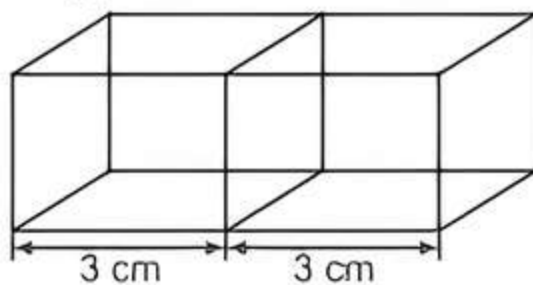
   ## 💡 TiP
   When we combine two cubes, then length of new cuboid is twice the length of cube's edge, but width and height remains same.

   Here, length $l = 2 \times 3 = 6$ cm, $b = 3$ cm and $h = 3$ cm.
   $\therefore$ The surface area of the resulting cuboid
   $$= 2(lb + bh + hl)$$
   $$= 2(6 \times 3 + 3 \times 3 + 3 \times 6)$$
   $$= 2(18 + 9 + 18)$$
   $$= 2(45) = 90 \text{ cm}^2$$
5. (d) Let 'r' be the radius of solid hemisphere.
   $\therefore$ Its diameter $(d) = 2 \times r$
   $$\Rightarrow \qquad r = \frac{d}{2}$$
   So, total surface area of solid hemisphere
   = CSA of hemisphere + Area of circular base
   $$= 2\pi r^2 + \pi r^2 = 3\pi r^2$$
   $$= 3\pi\left(\frac{d}{2}\right)^2 = \frac{3}{4}\pi d^2$$
6. (a) If two solid hemispheres of same base radius 'r' are joined together along their bases, then new solid becomes a solid sphere.
   CSA of new solid (sphere)
   = 2 × CSA of a hemisphere of same base radius 'r'
   $$= 2 \times 2\pi r^2 = 4\pi r^2$$
7. (b) Given, h and r be the height and radius of cylinder. If we cut the cylinder from the middle part, then height of remaining cylinder will be $\frac{h}{2}$.

   $\therefore$ The volume of the remaining cylinder
   $$= \pi r^2\left(\frac{h}{2}\right) = \frac{\pi r^2 h}{2}.$$
8. (a) Let the radius and height of a right circular cylinder be r and h respectively.
   Given, height of the cylinder $(h) = 14$ cm
   and curved surface area of a cylinder = 88 cm^2
   $$\therefore \qquad 2\pi rh = 88$$
   $$\Rightarrow \qquad 2 \times \frac{22}{7} r \times 14 = 88$$
   $$\Rightarrow \qquad r = \frac{88 \times 7}{2 \times 22 \times 14} = 1 \text{ cm}$$
   So, diameter of its circular base = $2r$
   $$= 2 \times 1 = 2 \text{ cm}$$
9. (d) Given, radius of a cone $(R) = r$
   and height of a cone $(h) = 3r$
   $\therefore$ Volume of a cone $= \frac{1}{3}\pi R^2 h$
   $$= \frac{1}{3}\pi r^2(3r)$$
   $$= \pi r^3$$
10. (b) Let the slant height of the cone be h.
    Given, radius of a cone $(r) = 7$ cm
    and curved surface area of a cone = 550 cm^2
    $$\therefore \qquad \pi rl = 550$$
    $$\Rightarrow \qquad \frac{22}{7} \times 7 \times l = 550$$
    $$\Rightarrow \qquad l = \frac{550}{22} = 25 \text{ cm}$$
    So, slant height of the cone is 25 cm.
11. (d) Given, base area of the cone = 156 cm^2
    and height of the vertical cone $(h) = 8$ cm
    $\therefore$ Volume of the right circular cone
    $$= \frac{1}{3} \times \text{base area} \times \text{height}$$
    $$= \frac{1}{3} \times 156 \times 8 = 52 \times 8 = 416 \text{ cm}^2$$
12. (b) The maximum height of cone
    = Height of the cylinder
    = 15 cm
13. (b) Let r_1 and r_2 be the radius of two spheres. Then
    $$\frac{\text{Volume of first sphere}}{\text{Volume of second sphere}} = \frac{64}{27}$$

$$\Rightarrow \quad \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \left(\frac{4}{3}\right)^3$$

$$\Rightarrow \quad \left(\frac{r_1}{r_2}\right)^3 = \left(\frac{4}{3}\right)^3$$

$$\Rightarrow \quad \frac{r_1}{r_2} = \frac{4}{3}$$

$$\therefore \text{ Ratio of two surface areas } = \frac{4\pi r_1^2}{4\pi r_2^2} = \left(\frac{r_1}{r_2}\right)^2$$

$$= \left(\frac{4}{3}\right)^2 = \frac{16}{9}$$

14. (a) Given, base diameter of a cylinder, $d = 1.6$ m and height, $h = 20$ cm.

$$\therefore \text{ Radius of cylinder, } r = \frac{d}{2} = \frac{1.6}{2} \text{ m}$$

$$= 0.8 \text{ m} = 80 \text{ cm}$$

So, the ratio of lateral surface area to the total surface

area of a cylinder $= \dfrac{2\pi r h}{2\pi r (h+r)} = \dfrac{h}{(h+r)}$

$$= \frac{20}{(20+80)} = \frac{20}{100} = \frac{1}{5}$$

$\therefore$ Required ratio is $1 : 5$.

15. (a) Given, radii and heights of two cylinders are in

ratio $\dfrac{r_1}{r_2} = \dfrac{3}{5}$ and $\dfrac{h_1}{h_2} = \dfrac{2}{3}$

$\therefore$ The ratio of two curved surface areas of two

cylinders $= \dfrac{2\pi r_1 h_1}{2\pi r_2 h_2} = \left(\dfrac{r_1}{r_2}\right)\left(\dfrac{h_1}{h_2}\right) = \dfrac{3}{5} \times \dfrac{2}{3} = \dfrac{2}{5}$

$\therefore$ Required ratio is $2 : 5$.

16. (b)

TiP
If solid ball is exactly fitted inside the cubical box of side 'a' unit. Then the diameter of the solid ball will be 'a' unit.

Here, diameter of the ball is 2 cm. So, its radius will be 1 cm.
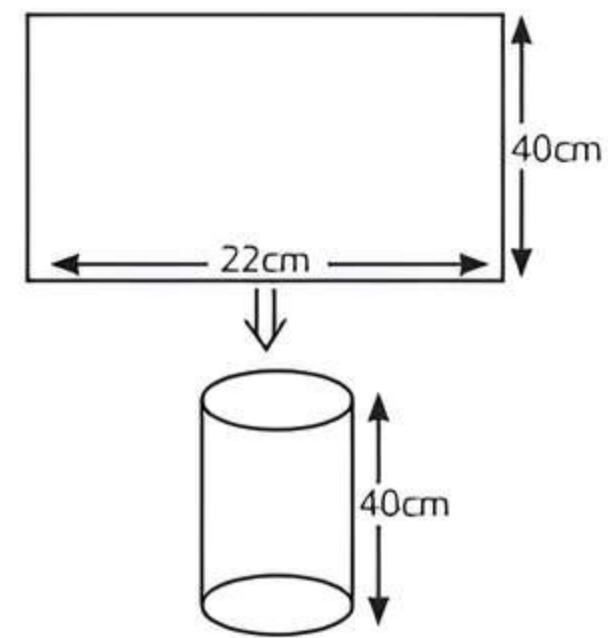
$\therefore$ The volume of the ball $= \dfrac{4}{3}\pi r^3 = \dfrac{4}{3} \times \pi (1)^3$

$$= \frac{4}{3}\pi \text{ cm}^3$$

17. (b) From the figure, it is clear that total height of a cylinder is $2H$ and radius of a cylinder is R.

$\therefore$ Total surface area of cylinder

$$= 2\pi R(R + 2H)$$
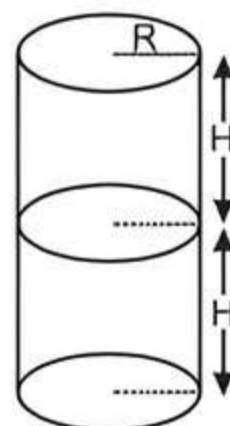
18. (a) Let r be the radius of so formed cylinder.

$\therefore$ Circumference of cylinder = breadth of rectangular sheet

TiP
When a rectangular sheet is rolled along their length, then a hollow cylinder is form. In which,
- *Height of cylinder = length of rectangular sheet.*
- *Circumference of cylinder = breadth of rectangular sheet*

$$\Rightarrow \quad 2\pi r = 22$$

$$\Rightarrow \quad 2 \times \frac{22}{7} \times r = 22$$

$$\Rightarrow \quad r = \frac{7}{2} = 3.5 \text{ cm}$$

So, radius of cylinder is 3.5 cm.

19. (b) Given radius of cone '$R = r$

$\therefore$ diameter of cone $= 2r$

So, height of the cone $(h) = 2r$ (By given condition)

Radius of hemisphere $(R') = r$ (By given condition)

$\therefore$ Volume of the solid = volume of cone + volume of hemisphere

$$= \frac{1}{3}\pi R^2 h + \frac{2}{3}\pi (R')^3$$

$$= \frac{1}{3}\pi r^2 (2r) + \frac{2}{3}\pi r^3 = \frac{4}{3}\pi r^3.$$

20. (b) Given radius of cone and hemisphere is $r = 3$ cm and height of cone is $h = 4$ cm.

TiP
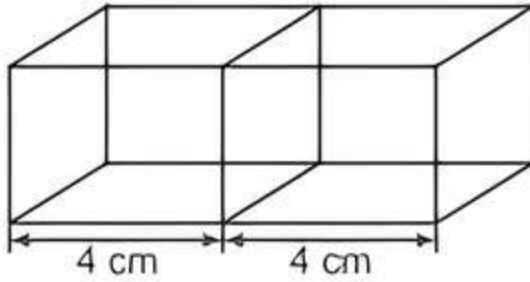*The slant height of a cone is $l = \sqrt{r^2 + h^2}$.*

Now, slant height of a cone, $l = \sqrt{r^2 + h^2}$

$$= \sqrt{(3)^2 + (4)^2}$$

$$= \sqrt{9 + 16} = \sqrt{25}$$

$$= 5 \text{ cm}$$

$\therefore$ The surface area of given figure

= Curved surface area of cone + curved surface area of hemisphere

$$= \pi r l + 2\pi r^2 = \pi r (l + 2r)$$

$$= \pi \times 3 (5 + 2 \times 3) = 3\pi (5 + 6) = 33\pi \text{ cm}^2$$

21. (a) **Assertion (A):** Given edge of a cube is 4 cm.

4 cm   4 cm

TiP

When we combine two cubes, then length of new cuboid is twice the original edge, but width and height remains same.

Here, length, $l = 2 \times 4 = 8$ cm.
width, $b = 4$ cm
and height, $h = 4$ cm

∴ The surface area of resulting cuboid
$$= 2\,(lb + bh + hl)$$
$$= 2\,(8 \times 4 + 4 \times 4 + 4 \times 8)$$
$$= 2\,(32 + 16 + 32) = 2 \times 80 = 160 \text{ cm}^2$$

So, Assertion (A) is correct.

Reason (R): It is true to say that length of the combined cubes is $2a$ cm.

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

22. (a) **Assertion (A):** Let r_1 and r_2 be the radii of two cones and h_1 and h_2 be the heights of two cones. Then
$$\frac{r_1}{r_2} = \frac{2}{3} \qquad \qquad \ldots(1)$$

Now, ratio of two volumes is $\dfrac{V_1}{V_2} = \dfrac{1}{3}$

$\Rightarrow$
$$\frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2} = \frac{1}{3} \quad \Rightarrow \quad \left(\frac{r_1}{r_2}\right)^2 \times \left(\frac{h_1}{h_2}\right) = \frac{1}{3}$$

$\Rightarrow$
$$\left(\frac{2}{3}\right)^2 \times \left(\frac{h_1}{h_2}\right) = \frac{1}{3} \qquad (\because \text{ from eq. (1)})$$

$\Rightarrow$
$$\frac{h_1}{h_2} = \frac{1}{3} \times \frac{9}{4} = \frac{3}{4}$$

So, Assertion (A) is true.

Reason (R): It is true that volume of any cone can be determined by the formula
$$V = \frac{1}{3}\pi r^2 h$$

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

23. (a) **Assertion (A):** We know that, the surface area of combined figure is the sum of all the curved surface of the individual figures.

TSA of the toy = CSA of hemisphere + CSA of the cone.

So, Assertion (A) is true.

Reason (R): It is also a true statement because any combined figure is obtained by fixing the plane surfaces of individual figures.

So, Reason (R) is true.

Hence, both Assertion (A) and Reason (R) is true and Reason (R) is the correct explanation of Assertion (A).

24. (d) **Assertion (A):** If the radius of a cone is halved and volume is not changed, then height is not remain same.

So, Assertion (A) is false.

Reason (R): Let V_1 and V_2 be the volume of two cones. Then
$$\frac{V_1}{V_2} = \frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2}$$
$$\frac{V_1}{V_2} = \frac{r^2 h_1}{\left(\frac{r}{2}\right)^2 h_2}$$

But V_1 and V_2 are equal

∴
$$r^2 h_1 = \left(\frac{r}{2}\right)^2 h_2$$

$\Rightarrow$
$$h_1 = \frac{1}{4} h_2$$

$\Rightarrow$
$$4h_1 = h_2$$

So, Reason (R) is true.

Hence, Assertion (A) is false but Reason (R) is true.

25. (b) **Assertion (A):** Let V_1 and V_2 be the volumes of two spheres. Also, let r_1 and r_2 be the radius of two spheres respectively. Then
$$\frac{V_1}{V_2} = \frac{216}{125}$$

$\Rightarrow$
$$\frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \left(\frac{6}{5}\right)^3$$

$\Rightarrow$
$$\left(\frac{r_1}{r_2}\right)^3 = \left(\frac{6}{5}\right)^3$$

$\Rightarrow$
$$\frac{r_1}{r_2} = \frac{6}{5}$$

∴ The ratio of surface areas of two spheres is
$$\frac{S_1}{S_2} = \frac{4\pi r_1^2}{4\pi r_2^2} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{6}{5}\right)^2 = \frac{36}{25}$$

Reason (R): It is also a true statement.

Hence, both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

26. diagonal

27. Let r be the radius of sphere and 'a' be the side of a cube.

Given that,
Surface area of sphere = Surface area of a cube

∴
$$4\pi r^2 = 6\,(a)^2$$

$\Rightarrow$
$$\frac{r^2}{a^2} = \frac{6}{4\pi}$$

$\Rightarrow$
$$\frac{r}{a} = \frac{\sqrt{6}}{2\sqrt{\pi}}$$

Now, $\dfrac{\text{Volume of sphere}}{\text{Volume of cube}} = \dfrac{\frac{4}{3}\pi r^3}{(a)^3}$

$= \dfrac{4}{3}\pi \times \left(\dfrac{r}{a}\right)^2 \left(\dfrac{r}{a}\right)$

$= \dfrac{4}{3}\pi \times \dfrac{6}{4\pi} \times \left(\dfrac{\sqrt{6}}{2\sqrt{\pi}}\right) = \sqrt{\dfrac{6}{\pi}}$

$= \sqrt{6} : \sqrt{\pi}$

28. Given, volume of a sphere, $V = 4851 \text{ m}^3$
Let r be the radius of sphere.

$\because \qquad V = 4851 \text{ m}^3$

$\Rightarrow \qquad \dfrac{4}{3}\pi r^3 = 4851 \Rightarrow \dfrac{4}{3}\times\dfrac{22}{7}\times r^3 = 4851$

$\Rightarrow \qquad r^3 = \dfrac{4851\times 3\times 7}{4\times 22} = \dfrac{441\times 21}{8}$

$\Rightarrow \qquad r^3 = \left(\dfrac{21}{2}\right)^3 \Rightarrow r = \dfrac{21}{2}\text{ cm}$

$\therefore$ Surface area of sphere $= 4\pi r^2 = 4\times\dfrac{22}{7}\times\left(\dfrac{21}{2}\right)^2$

$= 1386 \text{ cm}^2$

29. Let l, b and h be the length, breadth and height of a cuboid.

Then, $\qquad x = lb$, $y = bh$ and $z = hl$

Now $\qquad xyz = lb \times bh \times hl$

$\qquad\qquad xyz = l^2 b^2 h^2$

$\Rightarrow \qquad lbh = \sqrt{xyz}$

So, the volume of cuboid $= lbh = \sqrt{xyz}$.

30. $\dfrac{2}{3}\pi\,(R^3 - r^3)$ cubic units.

31. True

32. To find out the maximum volume of cone that can be carved out from hemisphere, the radius and height of cone will be r.

$\therefore$ The maximum volume of cone $= \dfrac{1}{3}\pi r^2 h$

$= \dfrac{1}{3}\pi r^2 \times r = \dfrac{\pi r^3}{3}$

Hence, given statement is false.

33. Let r_1 and r_2 be the radii of two spheres. Then ratio of volume of spheres is

$\dfrac{V_1}{V_2} = \dfrac{8}{27}$

$\Rightarrow \qquad \dfrac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \dfrac{8}{27} \Rightarrow \left(\dfrac{r_1}{r_2}\right)^3 = \dfrac{8}{27}$

$\Rightarrow \qquad \left(\dfrac{r_1}{r_2}\right)^3 = \left(\dfrac{2}{3}\right)^3 \Rightarrow \dfrac{r_1}{r_2} = \dfrac{2}{3} \qquad \ldots(1)$

$\therefore$ The ratio between their surface areas is

$\dfrac{S_1}{S_2} = \dfrac{4\pi r_1^2}{4\pi r_2^2} = \left(\dfrac{2}{3}\right)^2 = \dfrac{4}{9}.$

Hence, the given statement is true.

34. True, because in a cube, length, breadth and height are all equal.

35. Let original radius of sphere be r unit. Then surface area of hemisphere

$\qquad\qquad S_1 = 2\pi r^2$

If $\qquad\qquad r = 2r$, then

$\qquad\qquad S_2 = 2\pi\,(2r)^2$

$\qquad\qquad\quad = 8\pi r^2$

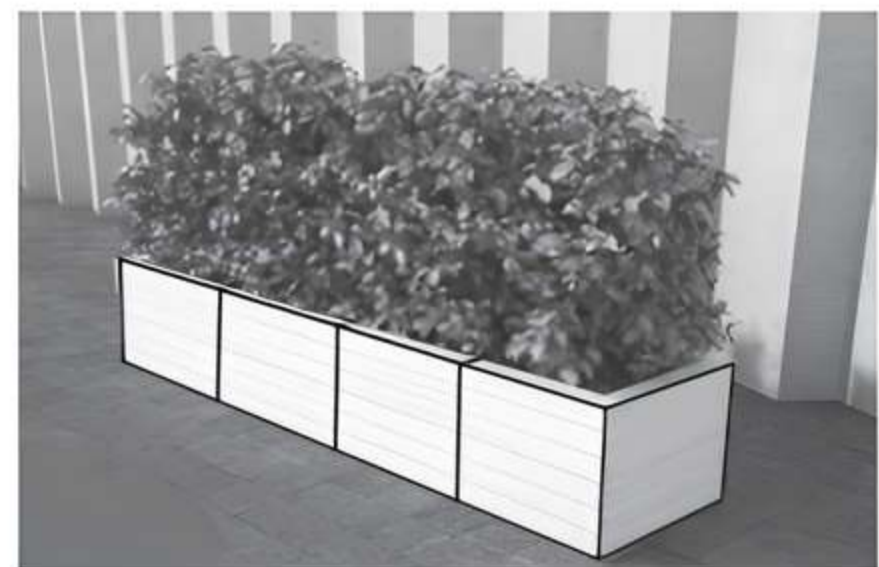$\qquad\qquad S_2 = 4(2\pi r^2)$

$\qquad\qquad S_2 = 4 S_1$

So, given statement is false.

💡 Case Study Based Questions ↘

Case Study 1

Avantika join four cubical open boxes of edge 20 cm each to make a pot for planting saplings of pudina in her kitchen garden. The saplings are cylindrical in shape with diameter 14.2 cm and height 11 cm.

Based on the above information, solve the following questions:

Q 1. If Avantika wants to paint the outer surface of the pot, then how much area she needs to paint?
a. 6400 cm^2 　　　　　 b. 5600 cm^2
c. 4200 cm^2 　　　　　 d. 2025 cm^2

Q 2. What is the volume of the pot formed?
a. 32000 cm^3 　　　　 b. 20250 cm^3
c. 40000 cm^3 　　　　 d. 10125 cm^3

Q 3. If Avantika decorates the four walls of the pot with coloured square paper of side 10 cm each, then how many pieces of papers would be required?
a. 120 　　　　　　　　 b. 54
c. 160 　　　　　　　　 d. 40

Q 4. Find the volume of 1 sapling.
a. 1742.75 cm^3 　　　 b. 4548.16 cm^3
c. 1764.08 cm^3 　　　 d. None of these

Q 5. If Avantika planted 4 saplings in the pot with some soil and compost up to the brim of the pot, then how much soil and compost are there in the pot?
a. 12612 cm^3 　　　　 b. 25029 cm^3
c. 21975 cm^3 　　　　 d. None of these

Solutions

1. Given, edge of each cubical box (a) = 20 cm
   ∴ Area to be painted = Area of $(24 - 6 - 4)$ i.e., 14 square faces
   $$= 14\, a^2 = 14\,(20)^2 = 5600 \text{ cm}^2$$
   So, option (b) is correct.

2. From figure,
   Length of the pot (l) = 20 cm
   Breadth of the pot (b) = 20 × 4 = 80 cm
   and height of the pot (h) = 20 cm
   Volume of pot = lbh = 20 × 80 × 20
   $$= 32000 \text{ cm}^3$$
   So, option (a) is correct.

3. Area of four walls = $2(l + b) \times h$
   $$= 2\,(20 + 80) \times 20 = 4000 \text{ cm}^2$$
   Given, side of coloured square paper = 10 cm
   Now, area of a square paper = $(10)^2 = 100 \text{ cm}^2$
   ∴ Number of pieces of paper required $= \dfrac{4000}{100} = 40$
   So, option (d) is correct.

4. Given, height of sapling (H) = 11 cm
   and diameter of sapling = 14.2 cm
   ∴ Radius of sapling $(R) = \dfrac{14.2}{2} = 7.1$ cm
   ∴ Volume of 1 sapling $= \pi R^2 H = \dfrac{22}{7} \times (7.1)^2 \times 11$
   $$= 1742.75 \text{ cm}^3$$
   So, option (a) is correct.

5. Total volume of pot = 32000 cm³
   Volume of 4 saplings = 1742.75 × 4 = 6971 cm³
   ∴ Volume of compost and soil = 32000 − 6971
   $$= 25029 \text{ cm}^3$$
   So, option (b) is correct.

Case Study 2

In some Muslim countries, eating in public during day-light hours in Ramadan is crime. The sale of alcohol becomes prohibited during Ramadan and alcohol is completely restricted in Ramadan Mela. At a Ramadan Mela, a stall keeper in one of the food stalls has a large cylindrical vessel of base radius 15 cm filled up to a height of 32 cm with orange juice. The juice is filled in small glasses, each small glass consist of a 6 cm long cylindrical portion attached to a hemisphere of radius 3 cm and sold for ₹ 15 each.
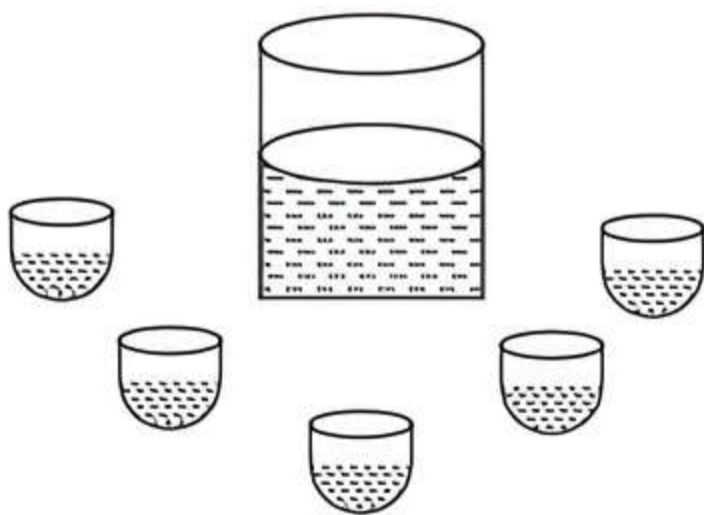
Based on the above information, solve the following questions:

Q 1. The volume of juice in the vessel is:
  a. 1200 π cm³   b. 4200 π cm³
  c. 5200 π cm³   d. 7200 π cm³

Q 2. The capacity of each small glass is:
  a. 72 π cm³   b. 42 π cm³
  c. 32 π cm³   d. 64 π cm³

Q 3. Number of glasses of juice that are sold:
  a. 10   b. 50   c. 100   d. 90

Q 4. How much money does the stall keeper receive by selling the juice completely?
  a. ₹ 1500   b. ₹ 750   c. ₹ 1250   d. ₹ 1750

Q 5. If 1/4 part of juice fall initially by stall keeper and then sold remaining juice for ₹ 25 each. How much money does the stall keeper receive by selling the remaining juice completely?
  a. ₹ 550   b. ₹ 1875   c. ₹ 650   d. ₹ 750

Solutions

1. Given that,
   Radius of the cylindrical vessel (R) = 15 cm
   and height of the cylindrical vessel (H) = 32 cm
   ∴ The volume of juice in the vessel
       = Volume of the cylindrical vessel
       = $\pi R^2 H$
       = π × 15 × 15 × 32 = 7200 π cm³
   So, option (d) is correct.

2. Given that,
   Height of the small glass (h) = 6 cm
   and radius of the small glass (r)
       = Radius of cylinder = Radius of hemisphere
       = 3 cm
   ∴ The capacity of juice in each glass can hold
       = Volume of each small glass
       = Volume of small cylinder
            + Volume of small hemisphere
   $$= \pi r^2 h + \frac{2}{3}\pi r^3$$
   $$= \frac{\pi r^2}{3}(3h + 2r)$$
   $$= \frac{\pi (3)^2}{3}(3 \times 6 + 2 \times 3)$$
   $$= 3\pi\,(18 + 6) = 72\pi \text{ cm}^3$$
   So, option (a) is correct.

3. The number of glasses of juice that are sold
   $$= \frac{\text{Volume of the vessel}}{\text{Volume of each glass}}$$
   $$= \frac{7200\pi}{72\pi} = 100$$
   So, option (c) is correct.

4. Amount received by the stall keeper
       = ₹ 15 × 100 = ₹ 1500
   So, option (a) is correct.

5. $\because$ The volume of juice in the vessel $= 7200\pi$ cm^3

$\therefore$ Volume of $\dfrac{1}{4}$ part of juice

$$= \dfrac{1}{4} \times 7200\pi = 1800\pi \text{ cm}^3$$

$\therefore$ Volume of remaining juice
$$= 7200\pi - 1800\pi = 5400\pi \text{ cm}^3$$

$\therefore$ The number of glasses of juice that are sold

$$= \dfrac{5400\pi}{72\pi} = 75$$

$\therefore$ Amount received by the stall keeper
$$= ₹ \, 25 \times 75 = ₹ \, 1875$$

So, option (b) is correct.

Case Study 3

A wooden toy is shown in the picture. This is a cuboidal wooden block of dimensions 14 cm × 17cm × 4 cm. On its top there are seven cylindrical hollows for bees to fit in. Each cylindrical hollow is of height 3 cm and radius 2 cm.

Based on the above information, solve the following questions: [CBSE 2023]

Q 1. Find the volume of wood carved out to make one cylindrical hollow.

Q 2. Find the lateral surface area of the cuboid to paint it with green colour.

Q 3. (a) Find the volume of wood in the remaining cuboid after carving out seven cylindrical hollows.

Or

(b) Find the surface area of the top surface of the cuboid to be painted yellow.

─────────┤ **Solutions** ├─────────

1. Given, height of the hollow cylinder $(h) = 3$ cm and radius of the hollow cylinder $(r) = 2$ cm
So, the volume of wood carved out to make one cylindrical hollow
$$= \text{volume of a hollow cylinder}$$
$$= \pi r^2 h$$
$$= \dfrac{22}{7} \times (2)^2 \times 3 = \dfrac{22}{7} \times 12 = \dfrac{264}{7} \text{cm}^3$$

2. Given, length of the cuboidal wooden block $(l) = 14$ cm, breadth of the cuboidal wooden block $(b) = 17$ cm and height of the cuboidal wooden block $(h) = 4$ cm

$\therefore$ Lateral surface area of the cuboid to paint it with green colour
$$= 2(l + b) \times h$$
$$= 2(14 + 17) \times 4$$
$$= 2 \times 31 \times 4 = 248 \text{ cm}^2$$

3. Volume of cuboidal wooden block $= l \times b \times h$
$$= 14 \times 17 \times 4$$
$$= 952 \text{ cm}^2$$
and volume of seven cylindrical hollows
$$= 7 \times \text{volume of a hollow cylinder}$$
$$= 7 \times \dfrac{264}{7} = 264 \text{cm}^3$$

$\therefore$ The volume of wood in the remaining cuboid after carving out seven cylindrical hollows.
= volume of cuboidal block − volume of seven cylindrical hollows.
$$= 952 - 264 = 688 \text{ cm}^3$$

Or

Surface area of the top surface of the cuboid
$$= l \times b = 14 \times 17 = 238 \text{ cm}^2$$
and curved surface area of seven circular region
$$= 7 \times \pi r^2 = 7 \times \dfrac{22}{7} \times 4 = 88 \text{cm}^2$$

$\therefore$ The surface area of the top surface of the cuboid to be painted yellow = surface area of the top surface of the cuboid − C.S.A of seven circular region
$$= 238 - 88 = 150 \text{ cm}^2.$$

Case Study 4

In a coffee shop, coffee is served in two types of cups. One is cylindrical in shape with diameter 7 cm and height 14 cm and the other is hemispherical with diameter 21 cm.

Based on the above information, solve the following questions: [CBSE 2023]

Q 1. Find the area of the base of the cylindrical cup.

Q 2. What is the curved surface area of the cylindrical cup?

Q 3. What is the capacity of the hemispherical cup?

Or

Find the capacity of the cylindrical cup.

─────────┤ **Solutions** ├─────────

1. Let r and h be the radius and height of the cylindrical cup respectively.
Given, diameter of the base $= 7$ cm

$\therefore$ Its radius $(r) = \dfrac{7}{2}$ cm

So, base area of the cylindrical cup $= \pi r^2$

$$= \frac{22}{7} \times \left(\frac{7}{2}\right)^2 = \frac{22}{7} \times \frac{49}{4} = \frac{77}{2}$$
$$= 38.5 \text{ cm}^2.$$

2. In cylindrical cup,

Given, radius $(r) = \frac{7}{2}$ cm and height $(h) = 14$ cm

∴ Curved surface area of the cylindrical cup
$$= 2\pi rh$$
$$= 2 \times \frac{22}{7} \times \frac{7}{2} \times 14 = 308 \text{ cm}^2.$$

3. Let R be the radius of the hemispherical cup.
   ∴ Given, diameter of hemispherical cup = 21 cm

   Its radius $(R) = \frac{21}{2}$ cm

   So, capacity of the hemispherical cup
   $$= \frac{2}{3}\pi R^3$$
   $$= \frac{2}{3} \times \frac{22}{7} \times \left(\frac{21}{2}\right)^3 = \frac{11 \times 21 \times 21}{2}$$
   $$= 2425.5 \text{ cm}^3.$$

   Or

   Given, height of the cylindrical cup $(h) = 14$ cm

   and its radius $(r) = \frac{7}{2}$ cm

   Capacity of the cylindrical cup
   $$= \pi r^2 h = \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times 14$$
   $$= 11 \times 7 \times 7 = 539 \text{ cm}^3.$$

Case Study 5

On diwali festival, a big company decided to gift his employees an electric kettle which was in a shape of cylinder and gift wrapped in the cubical box. The dimension of box is 20 cm × 15 cm × 30 cm and the radius and height of electrical kettle are 14 cm and 25 cm.

Based on the above information, solve the following questions:

Q 1. Find the volume of the box.

Q 2. Find the maximum length of rod that can be kept in the box.

Q 3. Find the area of the wrapping sheet that covers the box exactly.

Or

Find the total surface area of an electric kettle.

Solutions

1. Given, dimension of a box is $l = 20$ cm, $b = 15$ cm
   and $h = 30$ cm
   The volume of the box = lbh
   $$= 20 \times 15 \times 30 = 9000 \text{ cm}^3.$$
   Hence, volume of the box is 9000 cm³.

2.

> **TiP**
> The maximum length of rod that can be kept in the box, is equal to the length of diagonal of a cuboid.

∴ The maximum length of rod
   = length of diagonal of a cuboid
   $$= \sqrt{l^2 + b^2 + h^2} = \sqrt{(20)^2 + (15)^2 + (30)^2}$$
   $$= \sqrt{400 + 225 + 900} = \sqrt{1525}$$
   $$= 5\sqrt{61} \text{ cm}$$

Hence, a maximum length that can be kept in the box is $5\sqrt{61}$ cm.

3. The area of the wrapping sheet that covers the box is equal to the surface area of the box.
   ∴ Surface area of the box = $2(lb + bh + hl)$
   $$= 2(20 \times 15 + 15 \times 30 + 30 \times 20)$$
   $$= 2(300 + 450 + 600)$$
   $$= 2(1350) = 2700 \text{ cm}^2.$$
   Hence, the area of the wrapping sheet that covers the box exactly is 2700 cm².

   Or

   Given, radius and height of an electric kettle are $r = 14$ cm and $h = 25$ cm.
   ∴ The total surface area of an electric kettle
   = total surface area of cylinder
   $$= 2\pi r(h + r)$$
   $$= 2 \times 3.14 \times 14(25 + 14)$$
   $$= 87.92 \times 39 = 3428.88 \text{ cm}^2$$
   Hence, surface area of an electric kettle is 3428.88 cm².

> **Very Short Answer** Type Questions

Q 1. Volume and surface area of a solid hemisphere are numerically equal. What is the diameter of hemisphere? [CBSE 2017]

Q 2. If two cubes of edge 2 cm each are joined end to end, find the surface area of the resulting cuboid. [U. Imp.]

Q 3. A ball is exactly fit inside the cubical box of side 'a' unit. What is the volume of the ball? [U. Imp.]

Q 4. Two cubes have their volumes in the ratio 8 : 27. Find the ratio of their surface areas.

Q 5. A cone of radius 5 cm and slant height 13 cm, then find the height of cone.

Short Answer Type-I Questions ↘

Q 1. A solid is in the shape of a cone mounted on a hemisphere of same base radius. If the curved surface areas of the hemispherical part and the conical part are equal, then find the ratio of the radius and the height of the conical part. *[CBSE 2020]*

Q 2. Two cones with same base radius 8 cm and height 15 cm are joined together along their bases. Find the surface area of the shape formed.

Q 3. What will be the approximate volume of the largest right circular cone that can be cut out from a cube of edge 4.2 cm?

Q 4. The volume of a right circular cylinder with its height equal to the radius is $25\frac{1}{7}$ cm^3. Find the height of the cylinder. $\left[\text{Use } \pi = \frac{22}{7}\right]$. *[CBSE 2020]*

Q 5. From a solid right circular cylinder of height 14 cm and base radius 6 cm, a right circular cone of same height and same base radius is removed. Find the volume of the remaining solid. *[CBSE 2020]*

Q 6. Two types of water tankers are available in a shop at the same rate.
First one is in a conical form of diameter 3 m and height 5 m.
Second one is in the form of a cylinder of diameter 3 m and height 2 m.
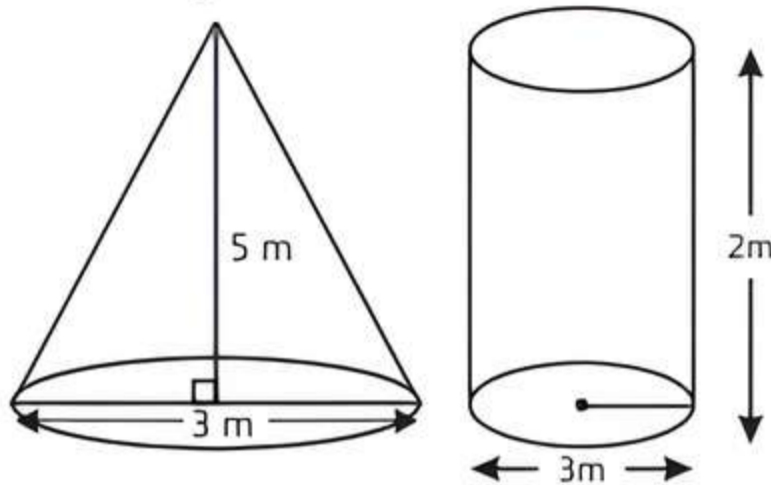
Out of the two, which tanker capacity (in litres) is more and how much?

Q 7. The curved surface area of a right circular cone is 12320 cm^2. If the radius of its base is 56 cm, then find its height. *[CBSE SQP 2022 Term-II]*

Short Answer Type-II Questions ↘

Q 1. A solid is in the shape of a right-circular cone surmounted on a hemisphere, the radius of each of them being 7 cm and the height of the cone is equal to its diameter. Find the volume of the solid.

[CBSE 2023]

Q 2. A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends (see the given figure). The length of the entire capsule is 14 mm and the diameter of the capsule is 5 mm.

Find its surface area. $\left[\text{Use } \pi = \frac{22}{7}\right]$

[NCERT EXERCISE; CBSE SQP 2023-24]
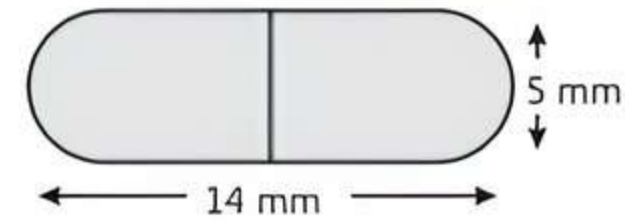
Q 3. From a solid cylinder of height 20 cm and diameter 12 cm, a conical cavity of height 8 cm and radius 6 cm is hollowed out. Find the total surface area of the remaining solid. *[CBSE 2023]*

Q 4. A heap of rice is in the form of a cone of base diameter 24 m and height 3.5 m. Find the volume of the rice. How much canvas cloth is required to just cover the heap? *[NCERT EXEMPLAR; CBSE 2018]*

Or

A heap or rice is in the form of a cone of base diameter 24 m and height $\frac{7}{2}$ m. Find the volume of rice. How much canvas cloth is required to just cover the heap? *[CBSE 2023]*

Q 5. Aditi went to a painter to get her two wooden toys painted.
Toy 1 is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius.
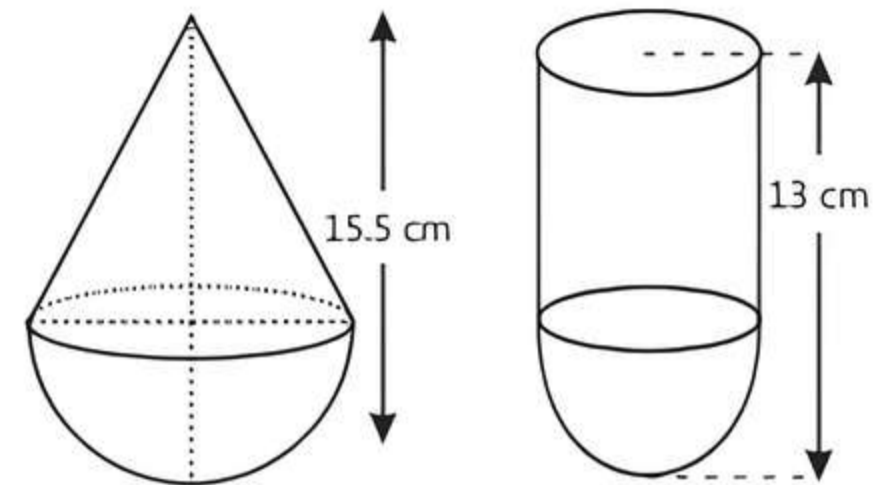
Toy 2 is in the form of a hemispherical bowl of diameter 7 cm mounted by a hollow cylinder.
These toys are painted from inside.
Out of the two, whose cost of painting at ₹ 1.50 per cm^2 is more and by how much?

Q 6. The sum of the radius of base and height of a solid right circular cylinder is 37cm. If the total surface area of the solid cylinder is 1628 cm^2, find the volume of the cylinder. $\left[\text{Use } \pi = \frac{22}{7}\right]$ *[CBSE 2016, 23]*

Q 7. From a solid right circular cylinder with height 12 cm and radius of the base 5 cm, a right circular cone of the same height and the same base radius is removed. Find the volume and total surface area of the remaining solid. [Use $\pi = 3.14$]
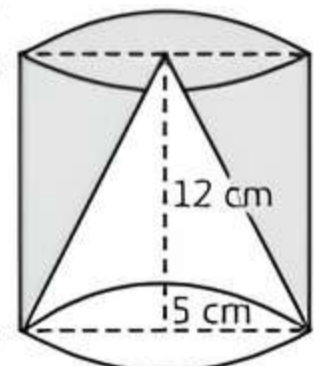
[CBSE 2015]

Q 8. A Juice seller was serving juice to his customers using glasses as shown in figure. The inner diameter of the cylindrical glass was 5 cm but bottom of the glass had a hemispherical raised portion which reduced the capacity of the glass. If the height of a glass was 10 cm, find the apparent and actual capacity of the glass. [Use $\pi = 3.14$].
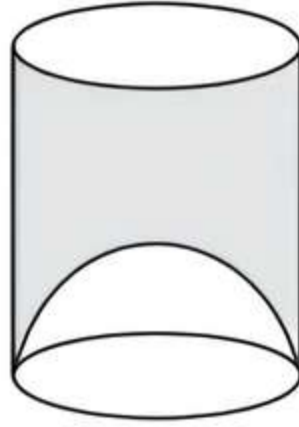
[NCERT EXERCISE; CBSE 2019]

Long Answer Type Questions ↘

Q 1. A solid wooden toy is in the form of a hemisphere surmounted by a cone of same radius. The radius of the hemisphere is 3.5 cm and the total wood used in the making of toy is $166\frac{5}{6}$ cm^3. Find the height of the toy. Also, find the cost of painting the hemispherical part of the toy at the rate of ₹ 10 per cm^2.

[CBSE 2022 Term-II]

Q 2. From a cuboidal solid metallic block of dimensions 15cm × 10 cm × 5 cm, a cylindrical hole of diameter 7 cm is drilled out. Find the surface area of the remaining block in cuboidal block.

Q 3. An iron pole consists of a cylinder of height 240 cm and base diameter 26 cm, which is surmounted by another cylinder of height 66 cm and radius 10 cm. Find the mass of the pole given that 1 cm^3 of iron has approximately 8 g mass. [Take, $\pi = 3.14$]

Q 4. A rocket is in the form of a right circular cylinder closed at the lower end and surmounted by a cone with the same radius as that of the cylinder. The diameter and height of the cylinder are 6 cm and 12 cm, respectively. If the slant height of the conical portion is 5 cm, then find the total surface area and volume of the rocket. [Use $\pi = 3.14$]

Q 5. A tent is in the shape of a cylinder surmounted by a conical top. If the height and radius of the cylindrical part are 3 m and 14 m respectively and the total height of the tent is 13.5 m, find the area of the canvas required for making the tent, keeping a provision of 26 m^2 of canvas for stitching and wastage. Also, find the cost of the canvas to be purchased at the rate of ₹ 500 per m^2.

[CBSE SQP 2023-24]

Q 6. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder as shown in the figure. If the height of the cylinder is 10 cm and its base is of radius 3.5 cm, find the total surface area of the article.   *[CBSE 2023]*

Q 7. A gulab jamun, contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 gulab jamuns, each shaped like cylinder with two hemispherical ends with length 5cm and diameter 2.8 cm.
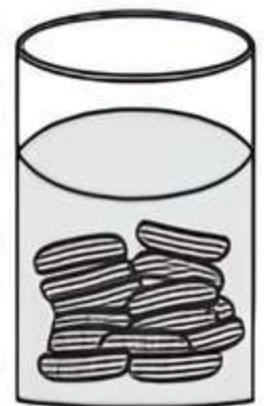
[CBSE SQP 2023-24, NCERT EXERCISE]

Q 8. A vessel is in the form of a hemispherical bowl surmounted by a hollow cylinder of same diameter. The diameter of the hemispherical bowl is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel. Also, find the volume of the vessel.   *[CBSE 2023]*

Solutions

Very Short Answer Type Questions

1. Let the radius of the hemisphere be r.

   $\therefore$ Volume of hemisphere $= \frac{2}{3}\pi r^3$

   and total surface area of solid hemisphere $= 3\pi r^2$
   According to the question,
   Volume of hemisphere

   $= $ Total surface area of hemisphere

   $\Rightarrow \quad \frac{2}{3}\pi r^3 = 3\pi r^2 \qquad \Rightarrow \quad 2r^3 = 9r^2$

   $\Rightarrow \quad r^2\left(r - \frac{9}{2}\right) = 0$

   $\Rightarrow \quad r = \frac{9}{2} \qquad\qquad (\because r \neq 0)$

   So, the diameter of hemisphere $= 2r$

   $= 2 \times \frac{9}{2} = 9$ units

   COMMON ERR⊙R
   Students take the formula of TSA of solid hemisphere as $2\pi r^2$ in haste but it is wrong. Students should understand the formula of TSA of hemisphere, which is $2\pi r^2 + \pi r^2 = 3\pi r^2$.

2. Given, edge of cube = 2 cm
   $\therefore$ Length of cuboid formed $(l) = 2 + 2 = 4$ cm
   Breadth of cuboid $(b) = 2$ cm
   and height of cuboid $(h) = 2$ cm.
   $\therefore$ Surface area of the resulting cuboid
   $= 2(lb + bh + hl)$

$= 2(4 \times 2 + 2 \times 2 + 2 \times 4)$
$= 2(8 + 4 + 8)$
$= 2 \times 20 = 40 \text{ cm}^2$

3. Given,
Diameter of the ball = Edge of the cubical box
= 'a' units
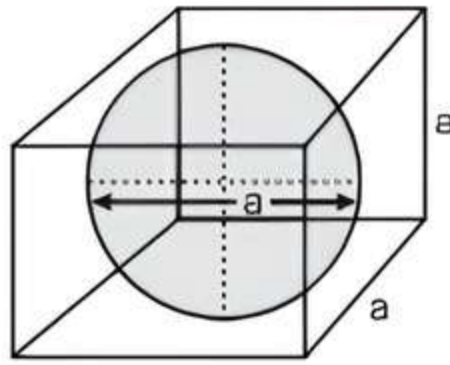
$\therefore$ Radius of the ball $(r) = \dfrac{a}{2}$ units

Hence, volume of the ball $= \dfrac{4}{3}\pi r^3$

$= \dfrac{4}{3}\pi\left(\dfrac{a}{2}\right)^3$

$= \dfrac{\pi a^3}{6}$ cubic units

4. Let 'a' and 'A' be the edges of two cubes. Then

$\dfrac{\text{Volume of first cube}}{\text{Volume of second cube}} = \dfrac{8}{27}$

$\Rightarrow \qquad \dfrac{a^3}{A^3} = \left(\dfrac{2}{3}\right)^3$

$\Rightarrow \qquad \dfrac{a}{A} = \dfrac{2}{3}$

$\therefore$ Ratio of surface areas of two cubes $= \dfrac{6a^2}{6A^2}$

$= \left(\dfrac{a}{A}\right)^2 = \left(\dfrac{2}{3}\right)^2 = \dfrac{4}{9} = 4:9$

5. Given radius of cone $r = 5$ cm
and slant height of cone $(l) = 13$ cm

$h = \sqrt{l^2 - r^2}$

$= \sqrt{(13)^2 - (5)^2} = \sqrt{169 - 25}$

$= \sqrt{144} = 12$ cm

Hence, height of cone is 12 cm.

Short Answer Type-I Questions

1. Let the height and slant height of the cone are h and l respectively.
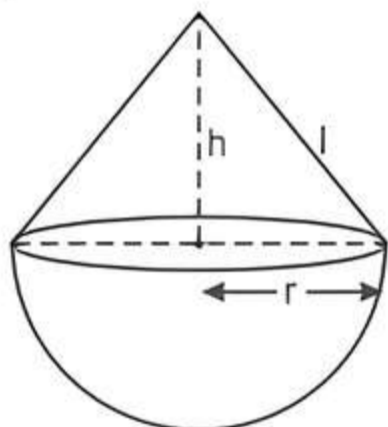
According to the question,
Radius of cone = Radius of hemisphere = r

Now, CSA of conical part = CSA of hemispherical part

$\Rightarrow \qquad \pi r l = 2\pi r^2$

$\Rightarrow \qquad \sqrt{r^2 + h^2} = 2r \qquad (\because l = \sqrt{h^2 + r^2})$

$\Rightarrow \qquad r^2 + h^2 = 4r^2 \Rightarrow h^2 = 3r^2$

$\Rightarrow \qquad \dfrac{r^2}{h^2} = \dfrac{1}{3} \Rightarrow \dfrac{r}{h} = \dfrac{1}{\sqrt{3}}$
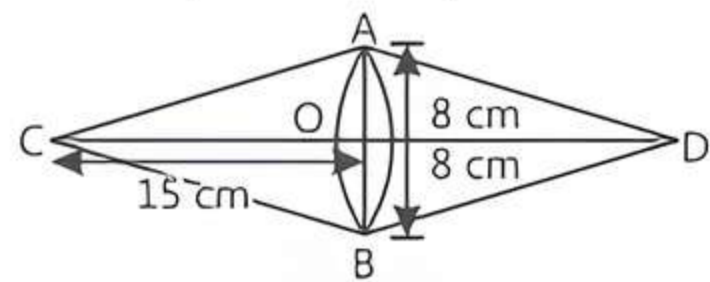
Hence, required ratio is $1 : \sqrt{3}$.

2. Given, radius of cone, $(r) = 8$ cm
and height of cone, $(h) = 15$ cm
Now, slant height of cone ABC,

$l = \sqrt{(OA)^2 + (OC)^2} = \sqrt{(8)^2 + (15)^2}$

$= \sqrt{64 + 225} = \sqrt{289} = 17$ cm
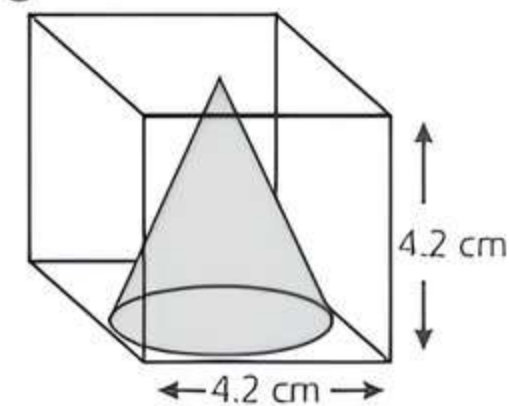

Now, surface area of the joined shape
= 2 × Curved surface area of cone ABC
= $2\pi r l$

$= 2 \times \dfrac{22}{7} \times 8 \times 17$

$= \dfrac{5984}{7} = 854.86 \text{ cm}^2$

3. Given, edge of a cube = 4.2 cm

From figure,
Height of cone (h) = Diameter of cone
= Edge of cone = 4.2 cm

$\therefore$ Radius of cone $(r) = \dfrac{4.2}{2} = 2.1$ cm

$\therefore$ Volume of the largest right circular cone

$= \dfrac{1}{3}\pi r^2 h = \dfrac{1}{3} \times \dfrac{22}{7} \times 2.1 \times 2.1 \times 4.2$

$= 22 \times 0.3 \times 0.7 \times 4.2 = 19.404 \text{ cm}^3$

4. Given, height of a right circular cylinder (h)
= radius of a right circular cylinder $(r) = x$ (say).

and volume of cylinder $= 25\dfrac{1}{7} = \dfrac{176}{7} \text{ cm}^3$

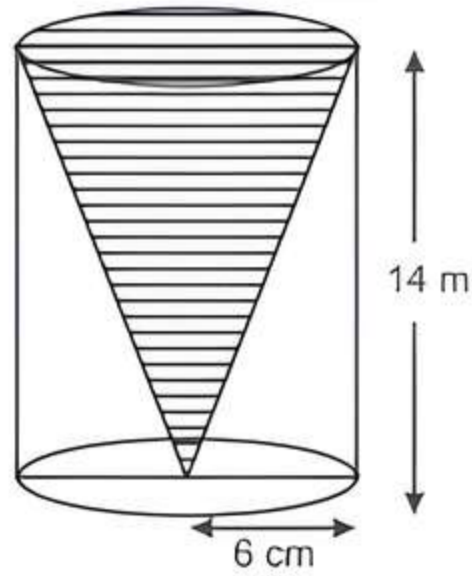$\therefore \qquad \pi r^2 h = \dfrac{176}{7}$

$\Rightarrow \qquad \dfrac{22}{7} \times x^2 \times x = \dfrac{176}{7}$

$\Rightarrow \qquad x^3 = 8 = 2^3$

$\Rightarrow \qquad x = 2$

Hence, required height is 2 cm.

5. Given that,

height of the cylinder (h) = height of the cone = 14 cm

and base radius of the cylinder
= base radius of the cone

$\Rightarrow$ $(r) = 6$ cm

$\therefore$ The volume of the remaining solid
= Volume of cylinder – Volume of cone

$= \pi r^2 h - \dfrac{1}{3}\pi r^2 h = \dfrac{2}{3}\pi r^2 h$

$= \dfrac{2}{3}\pi \times (6)^2 \times 14 = \dfrac{2}{3} \times \dfrac{22}{7} \times 36 \times 14 = 1056 \text{ cm}^3$

6. Given, radius of cone $(r) = \dfrac{3}{2}$ m and height of cone $(h) = 5$

$\therefore$ Volume of cone $= \dfrac{1}{3}\pi r^2 h$

$= \dfrac{1}{3} \times \dfrac{22}{7} \times \dfrac{3}{2} \times \dfrac{3}{2} \times 5$

$= 11.7857 \text{ m}^3$

$= 11785.7 \text{ L}$ $\qquad (\because 1 \text{m}^3 = 1000 \text{ L})$

Also, radius of cylinder $(R) = \dfrac{3}{2}$ m ;

and height of cylinder $(H) = 2$ m

$\therefore$ Volume of cylinder $= \pi R^2 H$

$= \dfrac{22}{7} \times \dfrac{3}{2} \times \dfrac{3}{2} \times 2$

$= 14.1428 \text{ m}^3$

$= 14142.8 \text{ L} \, (\because 1 \text{ m}^3 = 1000 \text{ L})$

Hence, capacity of cylindrical tanker is more by

$(14142.8 - 11785.7) = 2357.1 \text{ L}$

7. Let r and h be the radius and height of a cone.

Given, radius of cone $(r) = 56$ cm

and curved surface area of cone = 12320 cm²

$\therefore \qquad \pi r l = 12320$

$\Rightarrow \qquad \dfrac{22}{7} \times 56 \times l = 12320$

$\Rightarrow \qquad l = \dfrac{12320 \times 7}{22 \times 56} = \dfrac{86240}{22 \times 56} = 70$

$\Rightarrow \qquad \sqrt{r^2 + h^2} = 70 \qquad (\because l^2 = r^2 + h^2)$

$\Rightarrow \qquad (56)^2 + h^2 = (70)^2$

$\Rightarrow \qquad h^2 = 4900 - 3136$

$= 1764$

$\Rightarrow \qquad h = 42$ cm

Hence, height of a cone is 42 cm.

Short Answer Type-II Questions

1. Let r and h be the radius and height of the cone respectively.

Given, radius of cone = radius of hemisphere (r)

$= 7$ cm

and height of the cone (h) = diameter of cone

$= 2r = 2 \times 7 = 14$ cm

$\therefore$ Volume of the solid = Volume of cone + Volume of hemisphere

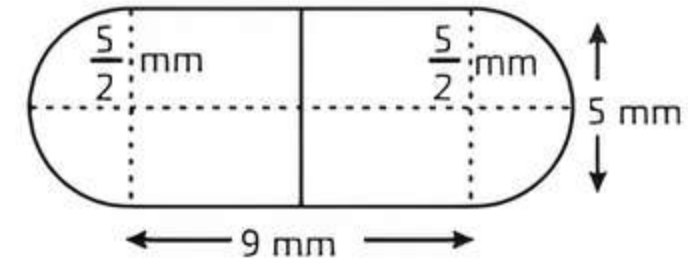$= \dfrac{1}{3}\pi r^2 h + \dfrac{2}{3}\pi r^3$

$= \dfrac{1}{3}\pi r^2 (h + 2r)$

$= \dfrac{1}{3} \times \dfrac{22}{7} \times (7)^2 \times (14 + 2 \times 7)$

$= \dfrac{22 \times 7}{3} \times (14 + 14) = \dfrac{22 \times 7 \times 28}{3}$

$= \dfrac{4312}{3} = 1437.33 \text{ cm}^3$

2.

From figure, radius of cylindrical part (r)
= Radius of hemispherical part (r)

$= \dfrac{\text{Diameter of the capsule}}{2} = \dfrac{5}{2}$ mm

Length of cylindrical part (h) = Length of the entire

capsule $- 2 \times r = 14 - 5 = 9$ mm

$\therefore$ Surface area of capsule

$= 2 \times$ CSA of hemispherical part
$+$ CSA of cylindrical part

$= 2 \times 2\pi r^2 + 2\pi r h$

$= 4\pi \left(\dfrac{5}{2}\right)^2 + 2\pi \left(\dfrac{5}{2}\right)(9) = 25\pi + 45\pi$

$= 70\pi \text{ mm}^2 = 70 \times \dfrac{22}{7} \text{ mm}^2 = 220 \text{ mm}^2$

Hence, the surface area of capsule is 220 mm².

3. Given, height of the solid cylinder $(h) = 20$ cm

height of the conical cavity (H)
$= 8$ cm
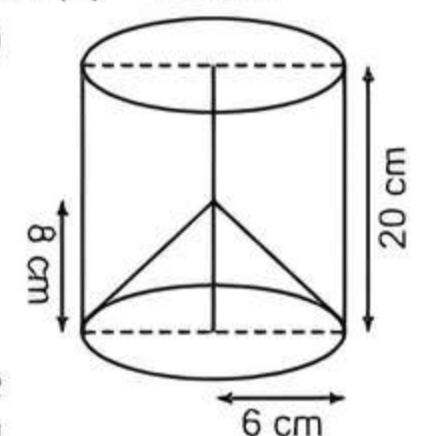
and radius of conical base
= radius of cylindrical base (r)
$= 6$ cm

$\therefore$ Total surface area of the remaining solid = Curved surface area of conical cavity + Area of circular base of cylindrical part

$= 2\pi rh + \pi rl + \pi r^2$

$= \pi r(2h + \sqrt{H^2 + r^2} + r)$

$= \dfrac{22}{7} \times 6 \times \left\{2 \times 20 + \sqrt{(8)^2 + (6)^2} + 6\right\}$

$= \dfrac{132}{7} \times \left\{40 + \sqrt{64 + 36} + 6\right\} = \dfrac{132}{7} \times \left\{40 + \sqrt{100} + 6\right\}$

$= \dfrac{132}{7} \times (40 + 10 + 6) = \dfrac{132 \times 56}{7} = 1056 \text{ cm}^2$

COMMON ERR⊙R

In problems related to surface area and volume, students write incorrect formula and get wrong result.

4. Given, diameter of the base of cone = 24 m

∴ Radius of the base of cone (r) = 24/2 = 12 m

Height of the cone (h) = 3.5 m

∵ A heap of rice is in the form of a cone.

∴ Volume of the rice = Volume of the conical heap

$= \dfrac{1}{3}\pi r^2 h = \dfrac{1}{3} \times \dfrac{22}{7} \times (12)^2 \times 3.5 \text{m}^3$

$= \dfrac{22}{21} \times 12 \times 12 \times 3.5 \text{m}^3 = 528 \text{m}^3$

Now, slant height of conical heap

$(l) = \sqrt{h^2 + r^2} = \sqrt{(3.5)^2 + 12^2}$

$= \sqrt{12.25 + 144} = \sqrt{156.25} = 12.5 \text{ m}$

TR!CK

Quantity of canvas cloth is equal to the surface area of heap to just cover it.

∴ Curved surface area of the heap = πrl

$= \dfrac{22}{7} \times 12 \times 12.5 \text{m}^2 = 471.43 \text{ m}^2$

Hence, the required canvas cloth to just cover the heap is 471.43 m².

5. **For Toy 1**

Given, radius of hemisphere (r) = Radius of conical portion

= 3.5 cm

Height of conical portion (h) = 15.5 − 3.5 = 12 cm

Slant height of conical portion $(l) = \sqrt{h^2 + r^2}$

$= \sqrt{(12)^2 + (3.5)^2} = \sqrt{144 + 12.25}$

$= \sqrt{156.25} = 12.5 \text{ cm}$

∴ TSA of toy = CSA of cone + CSA of hemisphere

$= \pi rl + 2\pi r^2 = \pi r(l + 2r)$

$= \dfrac{22}{7} \times 3.5 (12.5 + 2 \times 3.5)$

$= 11 \times 19.5 = 214.5 \text{ cm}^2$

So, Cost of painting the toy = ₹ (1.50 × 214.5)

= ₹ 321.75

For Toy 2

Given, radius of hemisphere (r) = Radius of cylinder

$= \dfrac{7}{2} \text{cm}$

= 3.5 cm

Height of cylinder (H) = 13 − 3.5 = 9.5 cm

∴ Inner surface area of toy = CSA of hemisphere + CSA of cylinder

$= 2\pi r^2 + 2\pi rH = 2\pi r(r + H)$

$= 2 \times \dfrac{22}{7} \times 3.5 (3.5 + 9.5)$

$= 22 \times 13 = 286 \text{ cm}^2$

So, Cost of painting the toy = ₹ (1.50 × 286)

= ₹ 429

Hence, cost of painting the second toy is more by (429 − 321.75) = ₹ 107.25

6. Let the radius and height of cylinder be 'r' and 'h' respectively.

TiP

Students should learn formulae of basic figures and do adequate practice and also concentrate to the correct calculation.

Given, $r + h = 37$ cm ....(1)

and total surface area of cylinder = 1628 cm²

⇒ $2\pi r(r + h) = 1628$

⇒ $2\pi r(37) = 1628$ (from eq. (1))

⇒ $2\pi r = \dfrac{1628}{37} = 44$

⇒ $2 \times \dfrac{22}{7} \times r = 44$

⇒ $r = \dfrac{44 \times 7}{2 \times 22} = 7 \text{ cm}$

∴ From eq. (1),

$7 + h = 37$

⇒ $h = 37 − 7 = 30 \text{ cm}$

Hence, the volume of cylinder = $\pi r^2 h$

$= \dfrac{22}{7} \times 7 \times 7 \times 30$

$= 4620 \text{ cm}^3$

7. Given, radius of cylinder (r) = Radius of cone (r) = 5 cm

Height of cylinder (h) = Height of cone (h) = 12 cm

∴ Slant height of the cone (l)

$= \sqrt{r^2 + h^2}$

$= \sqrt{5^2 + 12^2} = \sqrt{25 + 144}$

$= \sqrt{169} = 13 \text{ cm}$

So, Volume of the remaining solid

= Volume of the cylinder − Volume of the cone

$= \pi r^2 h - \dfrac{1}{3}\pi r^2 h$

$= \dfrac{2}{3}\pi r^2 h = \dfrac{2}{3} \times 3.14 \times 5^2 \times 12 \text{ cm}^3 = 628 \text{ cm}^3$

Now, total surface area of the remaining solid

= CSA of the cylinder + CSA of the cone + Area of the top of the cylinder

$= 2\pi rh + \pi rl + \pi r^2 = \pi r(2h + l + r)$

$$= 3.14 \times 5(2 \times 12 + 13 + 5)\,cm^2$$
$$= 3.14 \times 5 \times 42 = 659.4\,cm^2$$

Hence, volume of the remaining solid is 628 cm³ and its total surface area is 659.4 cm².

COMMON ERR!OR

Sometimes students make mistake in finding the surface area of the remaining solid. They take surface area of solid = surface area of cylinder − surface area of cone, but it is wrong. The right way of finding surface area = Curve surface area of cylinder + surface area of cone + Area of top of cylinder.

8. Given, height of the glass, $(h) = 10$ cm and inner diameter of the glass, $(d) = 5$ cm

∴ Inner radius of the glass, $(r) = \dfrac{d}{2} = \dfrac{5}{2} = 2.5$ cm

Now, apparent capacity of glass

$$= \text{Volume of cylinder} = \pi r^2 h$$
$$= 3.14 \times (2.5)^2 \times 10$$
$$= 3.14 \times 6.25 \times 10 = 196.25\,cm^3$$

Now, volume of hemispherical raised portion

$$= \frac{2}{3}\pi r^3 = \frac{2}{3} \times 3.14 \times (2.5)^3$$
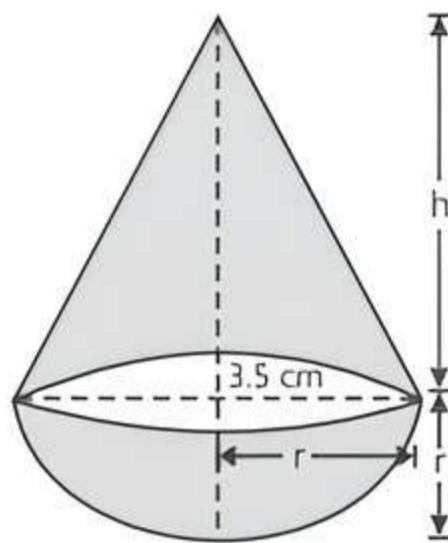$$= \frac{2}{3} \times 3.14 \times 15.625 = 32.71\,cm^3$$

∴ The actual capacity of the glass

$$= \text{Apparent capacity of glass} - \text{Volume of hemispherical raised portion}$$
$$= 196.25 - 32.71 = 163.54\,cm^3$$

Long Answer Type Questions

1. Let the height of the conical part be 'h' cm.

3.5 cm

Given, radius of cone (r) = radius of hemisphere
$$(r) = 3.5\,cm$$

∵ Volume of the wood used to make the toy

$$= 166\frac{5}{6}\,cm^3$$

∴ Volume of hemispherical part
$$\qquad + \text{Volume of the conical part}$$
$$= \frac{1001}{6}\,cm^3$$

TiP

Students should learn formulae of basic figures and do adequate practice and do correct calculations.

$$\Rightarrow \qquad \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h = \frac{1001}{6}$$

$$\Rightarrow \qquad \frac{1}{3}\pi r^2(2r + h) = \frac{1001}{6}$$
$$\Rightarrow \quad \frac{1}{3} \times \frac{22}{7} \times (3.5)^2\,(2 \times 3.5 + h) = \frac{1001}{6}$$
$$\Rightarrow \qquad 7 + h = \frac{1001}{6} \times \frac{3 \times 7}{22 \times (3.5)^2}$$
$$\Rightarrow \qquad 7 + h = 13$$
$$\therefore \qquad h = 13 - 7 = 6\,cm$$

So, the height of toy $= h + r = 6 + 3.5 = 9.5$ cm

Now, Area to be painted = CSA of the hemisphere

$$= 2\pi r^2$$
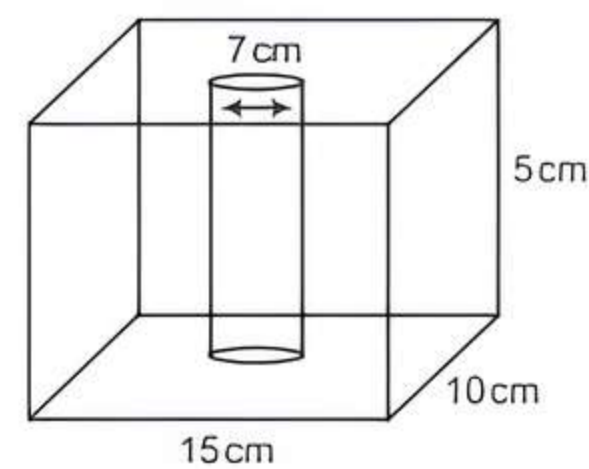$$= 2 \times \frac{22}{7} \times (3.5)^2\,cm^2 = 77\,cm^2$$

∵ Cost of painting 1 cm² area = ₹ 10

Hence, the cost of painting 77cm² area = ₹ (77×10)
$$= ₹\,770$$

2. In cuboidal block,
   length $l = 15$ cm
   breadth $b = 10$ cm
   and height $h = 5$ cm

∴ Surface area of the cuboidal block
$$= 2(lb + bh + hl)$$
$$= 2(15 \times 10 + 10 \times 5 + 5 \times 15)$$
$$= 2(150 + 50 + 75) = 2 \times 275 = 550\,cm^2.$$

Given diameter of cylindrical hole is 7 cm and its height $h = 5$ cm.

∴ Its radius $(r) = \dfrac{7}{2}$ cm

Curved surface area of cylindrical hole $= 2\pi rh$

$$= 2 \times \frac{22}{7} \times \frac{7}{2} \times 5 = 110\,cm^2$$

Area of two circular base of cylindrical hole

$$= \pi r^2 + \pi r^2 = 2 \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 = 77\,cm^2$$

∴ The surface area of the remaining block
$$= \text{Surface area of cuboidal block}$$
$$\quad + \text{CSA of cylindrical hole}$$
$$\quad - \text{area of two circular bases of cylindrical hole}$$
$$= 550\,cm^2 + 110\,cm^2 - 77\,cm^2 = 583\,cm^2.$$

3. Here, solid iron pole is a combination of two cylinders.
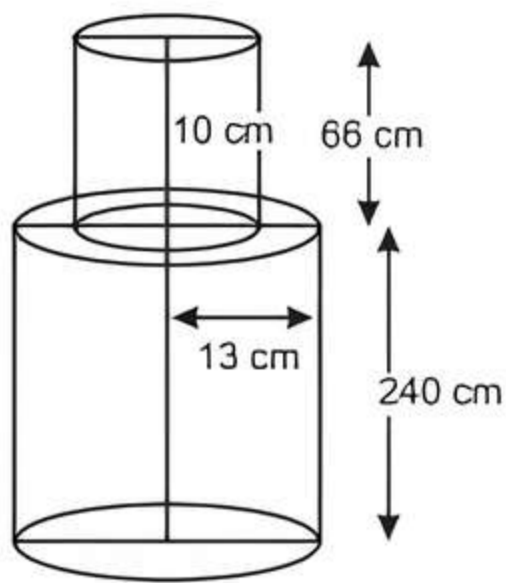   For first cylinder,
   Height $(h_1) = 240$ cm
   Base diameter $= 26$ cm

∴ Base radius $(r_1) = \dfrac{26}{2}$ cm $= 13$ cm

For second cylinder, height $(h_2) = 66$ cm
   Radius $(r_2) = 10$ cm

∵ Volume of cylinder $= \pi r^2 h$

∴ Total volume of iron pole = Volume of first cylinder
+ Volume of second cylinder

$= \pi r_1^2 h_1 + \pi r_2^2 h_2$

$= \pi (13)^2 \times 240 + \pi (10)^2 \times 66$

$= \pi [169 \times 240 + 100 \times 66]$

$= 3.14 (40560 + 6600) = 3.14 \times 47160$

$= 148082.4 \text{ cm}^3$

Hence, total mass of the iron pole

$= 148082.4 \times 8 \text{ g} = 1184659.2 \text{ g}$

(given, $1 \text{ cm}^3 \approx 8$ g)

$= \dfrac{1184659.2}{1000} \text{ kg}$

$= 1184.66 \text{ kg}$ $\left(\because 1\text{g} = \dfrac{1}{1000}\text{kg} \right)$

4. Given, rocket is the combination of a right circular cylinder and a cone.

Also, diameter of the cylinder = 6 cm
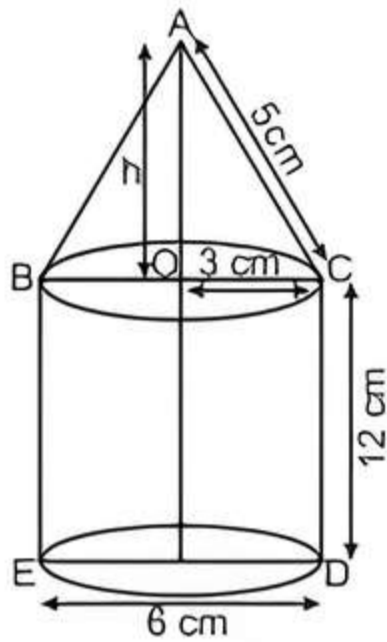
∴ Radius of the cylinder $(r) = \dfrac{6}{2} = 3$ cm

and height of the cylinder $(h) = 12$ cm

Now area of base of cylinder

$= \pi r^2 = 3.14 \times (3)^2 = 3.14 \times 9 = 28.26 \text{ cm}^2$.

∴ Volume of the cylinder $= \pi r^2 h = 3.14 \times (3)^2 \times 12$

$= 339.12 \text{ cm}^3$

and curved surface area $= 2\pi r h$

$= 2 \times 3.14 \times 3 \times 12$

$= 226.08 \text{ cm}^2$

Now, in right angled $\triangle AOC$, use Pythagoras theorem

$h = \sqrt{5^2 - 3^2} = \sqrt{25-9} = \sqrt{16} = 4$

∴ Height of the cone, $H = 4$ cm

and radius of the cone, $r = 3$ cm

Now, volume of the cone $= \dfrac{1}{3}\pi r^2 H = \dfrac{1}{3} \times 3.14 \times (3)^2 \times 4$

$= \dfrac{113.04}{3} = 37.68 \text{ cm}^3$

and curved surface area of cone
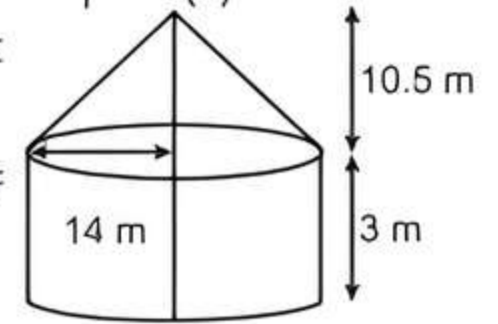
$= \pi r l = 3.14 \times 3 \times 5 = 47.1 \text{ cm}^2$

Hence, total volume of the rocket

= volume of cylinder + volume of cone

$= 339.12 + 37.68 = 376.8 \text{ cm}^3$

and total surface area of the rocket

= CSA of cone + CSA of cylinder
+ Area of base of cylinder

$= 47.1 + 226.08 + 28.26 = 301.44 \text{ cm}^2$

5. Given, height of the cylindrical part $(h) = 3$ m and radius of the cylindrical part $(r) = 14$ m

∴ Curved surface area of cylindrical part

$= 2\pi r h = 2 \times \dfrac{22}{7} \times 14 \times 3$

$= 264 \text{ m}^2$

Also, given total height of the tent $(h') = 13.5$ m

∴ Height of the conical part $(H) = h' - h$

$= 13.5 - 3 = 10.5$ m

and radius of conical part

= radius of cylindrical part $(r) = 14$ m

∴ Curved surface area of conical part

$= \pi r l = \pi r \sqrt{r^2 + H^2}$

$= \dfrac{22}{7} \times 14 \times \sqrt{(14)^2 + (10.5)^2}$

$= 44 \times \sqrt{196 + 110.25}$

$= 44 \times \sqrt{306.25} = 44 \times 17.5$

$= 770 \text{ m}^2$

∴ Total surface area of tent = CSA of cylindrical part
+ CSA of conical part $= 264 + 770 = 1034 \text{ m}^2$

∵ Provision for stitching and wastage $= 26 \text{ m}^2$

∴ Required area of canvas for making the tent

$= 1034 + 26 = 1060 \text{ m}^2$

So, cost of the canvas to be purchased

$=$ Rate $\times$ TSA $= ₹ 500 \times 1060 = ₹ 5,30,000$

6. Given, the height of the cylinder $h = 10$ cm and the base radius of cylinder $r = 3.5$ cm

Also, the base radius of hemisphere

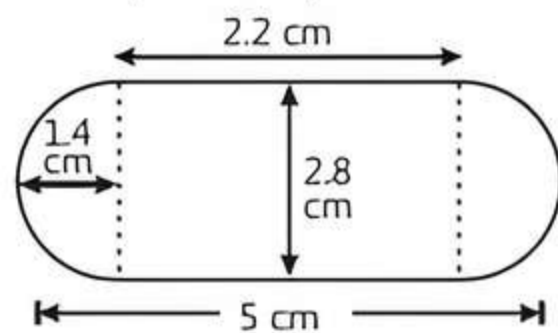= base radius of cylinder (r)

= 3.5 cm.

∴ Total surface area of the article = curved surface area of cylinder + 2 × curved surface area of hemisphere

$= 2\pi r h + 2 \times 2\pi r^2 = 2\pi r (h + 2r)$

$= 2 \times \dfrac{22}{7} \times 3.5 \times (10 + 2 \times 3.5)$

$= \dfrac{44}{7} \times 3.5 \times 17 = 374 \text{cm}^2$

7. It can be observed that radius (r) of cylindrical part
= Radius of hemispherical part ≈ 1.4 cm

Length (h) of cylindrical part
= 5 – 2 × length of hemispherical part
= 5– 2 × 1.4 = 5 –2.8 = 2.2 cm

Volume of one gulab jamun
= Volume of cylindrical part +2× volume of hemispherical part

$$= \pi r^2 h + 2 \times \frac{2}{3}\pi r^3 = \pi r^2 h + \frac{4}{3}\pi r^3$$

$$= \pi \times (1.4)^2 \times 2.2 + \frac{4}{3}\pi(1.4)^3$$

$$= \frac{22}{7} \times 1.4 \times 1.4 \times 2.2 + \frac{4}{3} \times \frac{22}{7} \times 1.4. \times 1.4 \times 1.4$$

= 13.552 + 11. 498 ≈ 25.05 cm³

∴ Volume of 45 gulab jamuns
= 45 × 25.05 ≈ 1127.25 cm³

So. volume of sugar syrup
≈ 30% of volume of 45 gulab jamuns

$$= \frac{30}{100} \times 1127.25 = 338.17 \text{cm}^3 ≈ 338 \text{ cm}^3$$

8. A cylinder is mounted on a hemisphere as shown in the figure.
Diameter of hemisphere (2r) ≈ 14 cm
Radius of hemisphere (r) = 7 cm
then radius of cylinder (r) = 7 cm

∴ Total height of the vessel is 13 cm which is equal to the sum of the height h of cylinder and radius r of hemisphere.


∴ $h + r ≈ 13$
$h + 7 = 13$
$h = 13 - 7 = 6$ cm

Height of cylinder $h ≈ 6$ cm
Then curved surface area of cylindrical part ≈ $2\pi rh$
and curved surface area of hemispherical part = $2\pi r^2$

∴ Total internal surface of vessel
$$= 2\pi rh + 2\pi r^2$$
$$= 2\pi r (h + r)$$
$$= 2 \times \frac{22}{7} \times 7(6 + 7)$$
$$= 2 \times 22 \times 13 \text{ cm}^2$$
$$= 572 \text{ cm}^2$$

and capacity of vessel = volume of hemispherical part + volume of cylindrical part
$$= \frac{2}{3}\pi r^3 + \pi r^2 h$$
$$= \frac{2}{3}\pi(7)^3 + \pi(7)^2 \times 6$$
$$= \frac{686}{3}\pi + 294\pi = \frac{1568}{3}\pi$$
$$= \frac{1568}{3} \times \frac{22}{7} = \frac{4928}{3} \text{ cm}^3$$

Therefore, the total inner surface area of vessel is 572 cm² and capacity is $\frac{4928}{3}$cm³ .
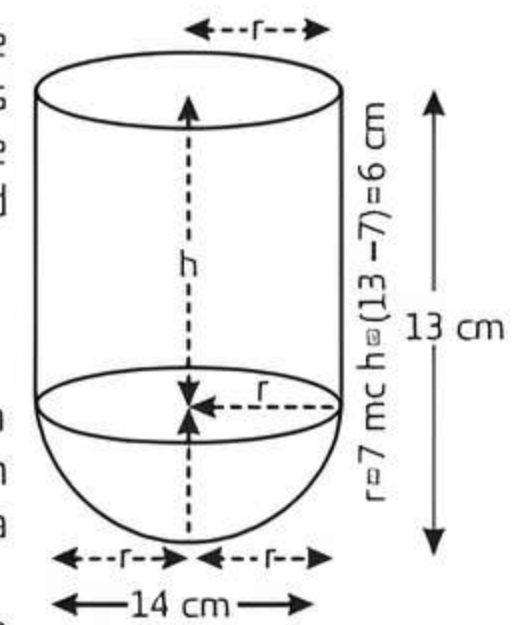
Chapter Test

Multiple Choice Questions

Q 1. Two different cylinders of same radius 7 cm but different heights 10 cm and 12 cm are joined along the base. The capacity of combined cylinder is: $\left[\text{Use } \pi = \frac{22}{7}\right]$

a. 3380 cm³   b. 3388 cm³
c. 3340 cm³   d. 3345 cm³

Q 2. Two cubes each of volume 8 cm³ are joined end to end. Then the surface area of resulting cuboid is:
a. 48 cm²   b. 44 cm²
c. 40 cm²   d. 50 cm²

Assertion and Reason Type Questions

Directions (Q. Nos. 3–4): *In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option:*
   a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)
c. Assertion (A) is true but Reason (R) is false.
d. Assertion (A) is false but Reason (R) is true.

Q 3. Assertion (A): A toy is in the form of a cone of radius 3.5 cm surmounted on a hemisphere of same radius. If the total height of the toy is 15.5 cm, then volume of the toy is 243.83 cm³. $\left[\text{Use } \pi = \frac{22}{7}\right]$

Reason (R): The volume of combined figure can be determined by sum of the volumes of individual solid figure.

Q 4. Assertion (A): Two identical cubes of each of volume 64 cm³ are joined together end to end. The surface area of the resulting cuboid is 350 cm².
**Reason (R):** The shape of a gilli, in the gilli-danda game is a combination of two cones and one cylinder.

Fill in the Blanks

Q 5. A solid ball is exactly fitted inside the cubical box of side *a*. The volume of the ball is ....

Q 6. If a cone of height 12 cm is mounted on a hemisphere of same radius 5 cm, then the height of the combined figure is ....

True/False

Q 7. Solid cone is exactly fitted inside a cubical box of side *a*. The volume of the cone is $\frac{\pi}{12}a^3$.

Q 8. The curved surface area of the given solid figure is $2\pi rh + \pi rl$.
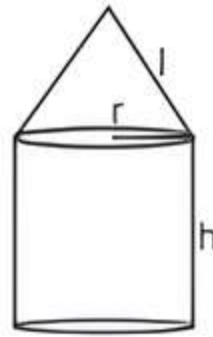
Case Study Based Question

Q 8. Ankita is studying in X standard. While helping her mother in kitchen, she saw rolling pin made of steel and empty from inner side, with two small hemispherical ends as shown in the figure.

Based on the above information, solve the following questions:

(i) Find the curved surface area of two identical cylindrical parts, if the diameter is 2.5 cm and length of each part is 5 cm.

(ii) Find the volume of big cylindrical part.

(iii) Find the volume of two hemispherical ends having diameter 2.5 cm.

OR

Find the curved surface area of two hemispherical ends having diameter 2.5 cm.

Very Short Answer Type Questions

Q 10. Two cubes have their volumes in the ratio 1 : 27. What is the ratio of their surface areas?

Q 11. A solid is composed of a cylinder with hemispherical ends. If the whole length of the solid is 104 cm and the radius of each hemispherical end is 7 cm, then find the cost of polishing its surface at the rate of ₹ 2 per dm². $\left[\text{Use } \pi = \frac{22}{7} \right]$

Short Answer Type-I Questions

Q 12. Two cones with same base radius 8 cm and height 15 cm are joined together along their bases. Find the surface area of the shape so formed.

Q 13. 50 circular plates, each of radius 7 cm and thickness 0.5 cm are placed one above another to form a solid right circular cylinder. Find the volume of the cylinder so formed.

Short Answer Type-II Questions

Q 14. A sphere of maximum volume is cut-out from a solid hemisphere of radius *r*. What is the ratio of the volume of the hemisphere to that of the cut-out sphere?

Q 15. A building is in the form of a cylinder surmounted by a hemispherical dome. The base diameter of the dome is equal to $\frac{2}{3}$ of the total height of the building. Find the height of the building, if it contains $67\frac{1}{21}$ m³ of air.

Long Answer Type Question

Q 16. A solid iron-pole having cylindrical portion 110 cm high and of base diameter 12 cm is surmounted by a cone 9 cm high. Find the mass of the pole, given that the mass of 1 cm³ of iron is 8 gm.

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